Quadratic Equation

Quadratic equations are the polynomial equations of degree 2 in one variable of type $f(x) = ax^2 + bx + c = 0$ where $a, b, c, \in R$ and $a \neq 0$. It is the general form of a quadratic equation where 'a' is called the leading coefficient and 'c' is called the absolute term of f (x). The values of x satisfying the quadratic equation are the roots of the quadratic equation (α, β).

Quadratic Equation Definition

A quadratic polynomial, when equated to zero, becomes a quadratic equation. In other terms, a quadratic equation is a second degree algebraic equation. The values of x satisfying the equation are called the roots of the quadratic equation.

General from: $ax^2 + bx + c = 0$

Here, $a, b, c, \in R$ and $a \neq 0$

Examples: $3x^2 + x + 5 = 0$, $-x^2 + 7x + 5 = 0$, $x^2 + x = 0$.

Quadratic Equation Formula

The solution or roots of a quadratic equation are given by the quadratic formula:

$$x = (\alpha, \beta) = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$$

Roots of Quadratic Equation

The values of variables satisfying the given quadratic equation are called their roots. In other words, $x = \alpha$ is a root of the quadratic equation f(x), if $f(\alpha) = 0$.

The real roots of equation $f(x) = 0$ are the x-coordinates of the points where the curve $y = f(x)$ intersects the x-axis.

- One of the roots of the quadratic equation is zero, and the other is $-b/a$ if $c = 0$
- Both the roots are zero if $b = c = 0$
- The roots are reciprocal to each other if $a = c$

What Is Discriminant?

The term $(b^2 - 4ac)$ in the quadratic formula is known as the discriminant of a quadratic equation. The discriminant of a quadratic equation reveals the nature of roots.

Some Examples

Example : Find the values of k for which the quadratic expression (x – k) (x – 10) + 1 = 0 has integral roots.

Solution:

The given equation can be rewritten as, $x^2 - (10 + k)x + 1 + 10k = 0$.

$D = b^2 - 4ac = 100 + k^2 + 20k - 40k = k^2 - 20k + 96 = (k - 10)^2 - 4$

The quadratic equation will have integral roots if the value of discriminant > 0, D is a perfect square, a = 1 and b and c are integers.

i.e., $(k - 10)^2 - D = 4$

Since the discriminant is a perfect square, the difference between two perfect squares in R.H.S will be 4 only when D = 0 and $(k - 10)^2 = 4$.

$\Rightarrow k - 10 = \pm 2$. Therefore, the values of k = 8 and 12.

Example : Find the values of k such that the equation p/(x + r) + q/(x – r) = k/2x has two equal roots.

Solution:

The given quadratic equation can be rewritten as:

$[2p + 2q - k]x^2 - 2r[p - q]x + r^2k = 0$

For equal roots, the discriminant (D) = 0, i.e., $b^2 - 4ac = 0$

Here, a = [2p + 2q – k], b = – 2r [p – q] and $c = r^2k$

$[-2r (p - q)]^2 - 4[(2p + 2q - k) (r^2k)] = 0$

$r^2(p - q)^2 - r^2k(2p + 2q - k) = 0$

Since $r \neq 0$, $(p - q)^2 - k(2p + 2q - k) = 0$

$k^2 - 2(p + q)k + (p - q)^2$

$k = 2(p+q) \pm \sqrt{[4(p + q)^2 - 4(p - q)]^2}/2 = -(p + q) \pm \sqrt{4pq}$

$\therefore$ The values of $k = (p + q) \pm 2\sqrt{pq} = (\sqrt{p} \pm \sqrt{q})^2$

Example : Find the quadratic equation with rational coefficients when one root is 1/(2 + √5).

Solution:

If the coefficients are rational, then the irrational roots occur in conjugate pairs. Therefore, if one root is $\alpha = 1/(2 + \sqrt{5}) = \sqrt{5} - 2$, then the other root will be $\beta = 1/(2 - \sqrt{5}) = -\sqrt{5} - 2$.

The sum of the roots $\alpha + \beta = -4$ and the product of roots $\alpha\beta = -1$.

Thus, the required equation is $x^2 + 4x - 1 = 0$.

Example : Form a quadratic equation with real coefficients when one of its roots is (3 – 2i).

Solution:

Since the complex roots always occur in pairs, the other root is 3 + 2i. Therefore, by obtaining the sum and the product of the roots, we can form the required quadratic equation.

The sum of the roots is

$(3 + 2i) + (3 - 2i) = 6$. The product of the root is $(3 + 2i) \times (3 - 2i) = 9 - 4i^2 = 9 + 4 = 13$.

Hence, the equation is $x^2 - Sx + P = 0$

Therefore, $x^2 - 6x + 13 = 0$ is the required quadratic equation.

EXERCISE

1. Equation of $(x+1)^2 - x^2 = 0$ has number of real roots equal to:

(a) 1

(b) 2

(c) 3

(d) 4

2. The roots of $100x^2 - 20x + 1 = 0$ is:

(a) 1/20 and 1/20

(b) 1/10 and 1/20

(c) 1/10 and 1/10

(d) None of the above

3. The sum of two numbers is 27 and product is 182. The numbers are:

(a) 12 and 13

(b) 13 and 14

(c) 12 and 15

(d) 13 and 24

4. If 1/2 is a root of the quadratic equation x^2-mx-5/4=0, then value of m is:

(a) 2

(b) -2

(c) -3

(d) 3

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, the other two sides of the triangle are equal to:

(a) Base=10cm and Altitude=5cm

(b) Base=12cm and Altitude=5cm

(c) Base=14cm and Altitude=10cm

(d) Base=12cm and Altitude=10cm

Answer Key

1. A

2. C

3. B

4. B

5. B

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